Understanding Uncertainty Maps in Vision with Statistical Testing

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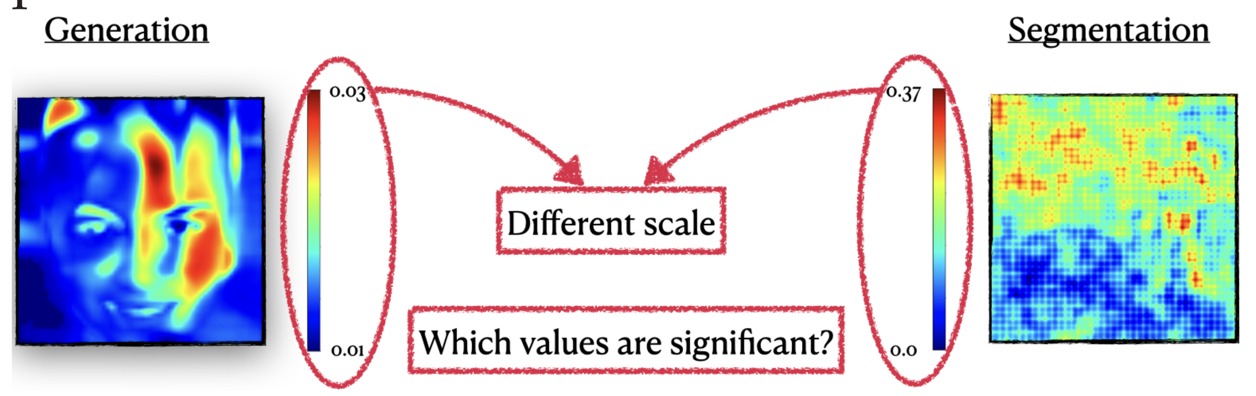
Overview

- We show how to select significant uncertainty in vision with help of Random Fields and Differential Equations theories.
- We develop a probabilistic framework: Warping Neural ODE (based on Neural ODE and Wasserstein distance), which enables learning a diffeomorphism between uncertainty maps and Gaussian Random Fields.
- In comparison to pixel-wise hypothesis testing (HT), our method accounts for pixel correlation, common in Vision.
- We show applications for different ways of generating uncertainty maps, like Bayesian Neural Networks, Variational Auto Encoders and MC dropout.
- We show applications in different vision settings, like generation, depth estimation and segmentation.

Motivation

- With probabilistic models we are able to create a notion of uncertainty in vision, e.g. pixel-wise variance. However, which values are statistically significant, given that scale depends on a nature of the problem?

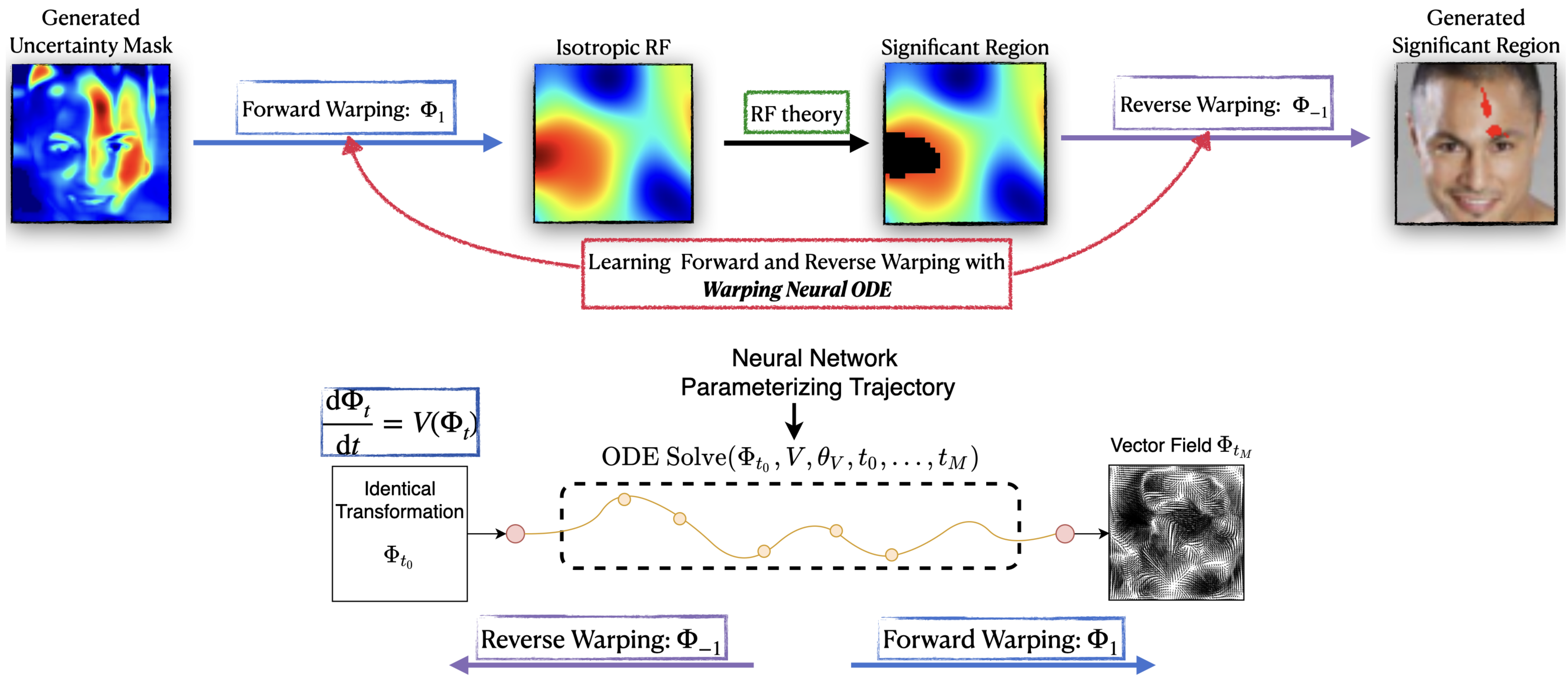


- Two common ways to select significant regions:

Types	Theoretical Support	Pixel Correlation	# of Tests
Pixel-wise HT	✓	✗	# of pixels
Quantile	✗	✗	1

- Is there a way to perform 1 HT with theoretical support and acknowledging pixel correlation? Yes!

Methods and Main Results



- We consider the pixel (voxel)-wise uncertainty map $M_{\mathbf{x}}$ as an RF over domain S, and derive the significance of uncertainty as Hypothesis Test:

$$\begin{bmatrix} H_0 : \forall s \in S, M_{\mathbf{x}}(s) = 0 \\ H_A : \exists s \in S, M_{\mathbf{x}}(s) \neq 0 \end{bmatrix}$$

- Gaussian Kinematic Formula

Theorem 1. *If F is GRRF (isotropic or non-isotropic), EEC is given as,*

$$\mathbb{P}(F_{\max} \geq u | H_0) \approx \mathbb{E}\{\phi(A_u)\} = \sum_{d=0}^{D} L_d(S, \Lambda(S))\rho_d(u),$$

where $F_{\max} = \max_{s \in S} M_{\mathbf{x}}(s)$ is a common test statistics.

- Uncertainties from probabilistic models are Gaussian Related Random Fields!
- Warping to GRF!

Theorem 2. *The domain S of the GRRF F can be warped via a one-to-one smooth transformation Γ to a domain S' without fundamentally changing the problem, namely:* $\mathbb{P}(\max_{s' \in S'} F(s') \geq t) = \mathbb{P}(\max_{s \in S} F(s) \geq t)$.

- Learning Warping $\Phi(S)$: two desired properties
  - $\Phi(S)$ is diffeomorphism: $\exists\ \Phi_1$ and Φ_{-1}.
  - the warped version is an isotropic GRF
- **Warping Neural ODE**: Using Neural ODE with Wasserstein distance, we parameterize a diffeomorphism from the Lie group, which allows to compute the *Reverse warping* through running DE reverse.

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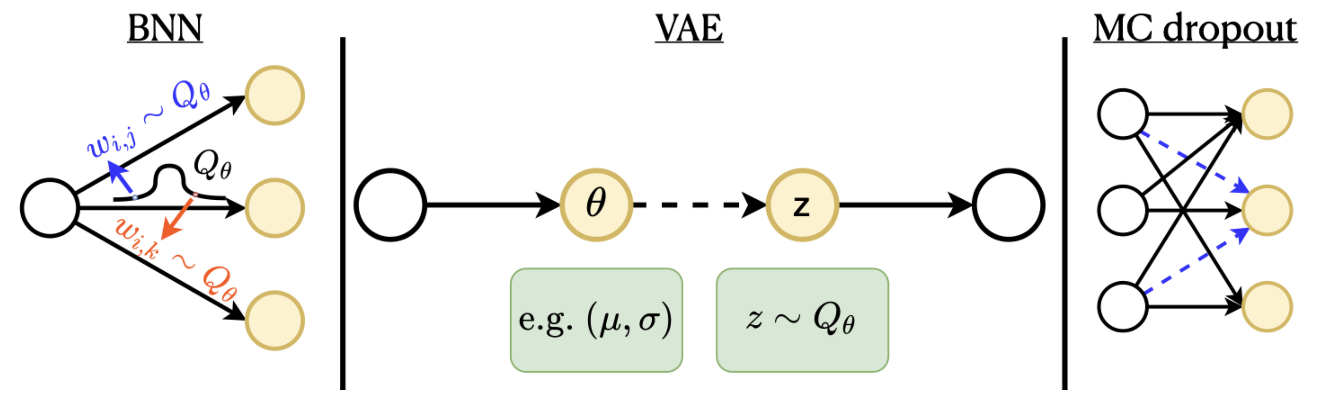
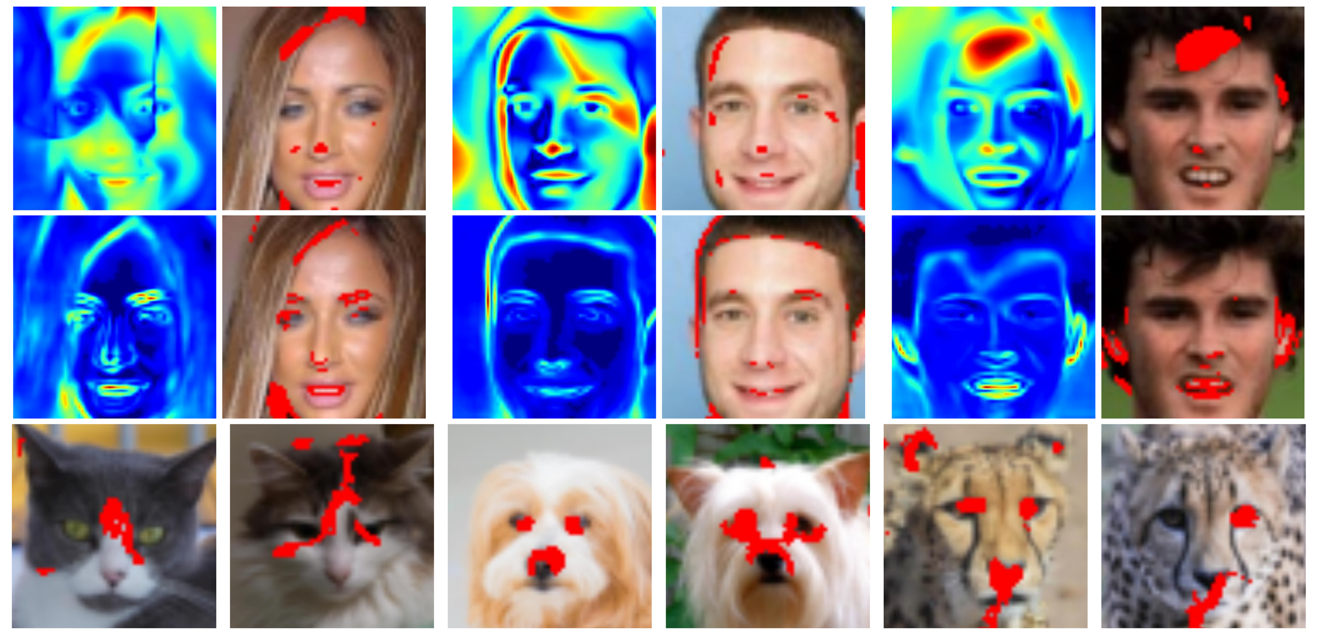
Detailed Results

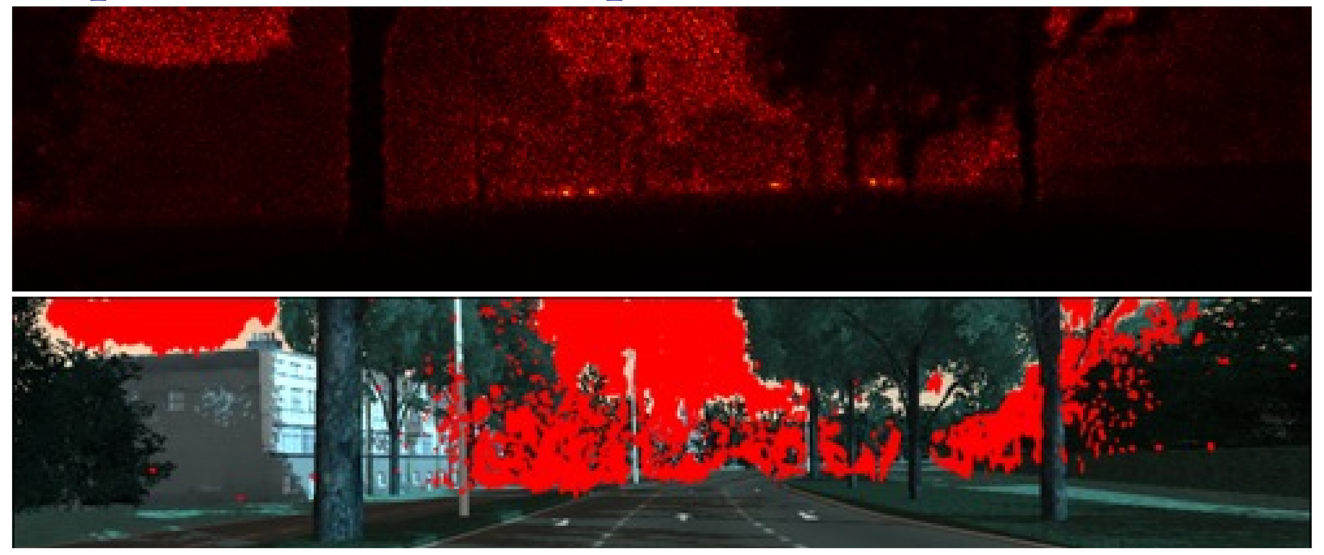
- **Ways to generate uncertainty mask**: we provide experiments of our method to detect significant regions of uncertainty mask, generated by common probabilistic models



- **Generation (VAE)**: Results on celebA (ResNet-18 and ResNet-50) and AFHQ (ResNet-18)



- **Depth estimation (Dropout)**: KITTI



- **Segmentation (Dropout)**: MS-COCO